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Particle trajectories in nonlinear gravity–capillary waves

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The trajectories of surface and subsurface particles of nonlinear gravity–capillary waves are calculated. Surface tension is found to have a small effect on the trajectories and drift velocity of pure gravity waves (down to 20 cm in length). On the other hand, pure capillary wave trajectories can be considerably altered even when the influence of gravity is small (waves of up to 2 cm in length). When the restoring forces are of comparable magnitude, some remarkable trajectories are possible, containing one or more subloops. Overall, the influence of increased surface tension is to increase the relative horizontal distance travelled by a particle, as well as the magnitude of the time-averaged drift velocity ratio at the surface and, as far as short waves are concerned, its penetration depth.

These results can have implications for steep waves where parasitic capillaries are generated and for observations of the wind-drift current.

1. Introduction

The trajectories of steady steep symmetric deep-water pure capillary waves were calculated recently in a paper by the present author (Hogan 1984). It was found possible to present the results in exact analytic form, up to and including the highest wave. In the steeper waves, the orbits were found to be neither circular nor closed. For the highest wave, a particle on the surface moves forward nearly eight wavelengths in the course of one orbit at a time-averaged drift velocity U equal to almost 90 % of the phase speed c of the wave.

In the present work, we investigate the influence of gravity on these results, considering the full range of possibilities from gravity-free to gravity-only propagation. We make use of computational results from earlier work (Hogan 1980, 1981) which dealt with the form of the wave profile. This is because no exact analytic solution has yet been found. The results show that gravity need only be quite weak for the trajectory properties of pure capillary waves to be influenced rather severely. At the other extreme, it appears that surface tension does not have a marked effect on the trajectories of pure gravity waves. When the two effects are comparable, however, some quite remarkable convoluted trajectories have been discovered. In general, the drift velocity ratio is curtailed in magnitude at the surface for all but the shortest waves.

The form of this paper is similar to Hogan (1984). Thus in §2 the problem of the propagation of free-surface nonlinear deep-water gravity–capillary waves is

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discussed. In §3 we derive formulae for the global properties of complete surface-particle trajectories and present results for selected gravity–capillary waves. In §4 the trajectories of surface and subsurface particles are presented. The drift-velocity ratio U/c is also presented as a function of the mean depth of fluid particles.

2. Gravity–capillary waves

The type of waves we consider are similar to those examined in Hogan (1984), except that we now include the effects of gravity. Thus we calculate the trajectories of particles in steady symmetric periodic nonlinear gravity–capillary waves which propagate on the surface of an incompressible inviscid infinitely deep fluid. The motion in the fluid is taken to be two-dimensional and irrotational and the wave is moving to the right with speed c. There is no motion at great depths. By moving in a frame of reference with the waves, we reduce the flow to a steady state. Cartesian axes are chosen with x measured horizontally to the right and y vertically upwards. The velocity potential is denoted by ϕ and the stream function by ψ, in the steady motion. The free surface corresponds to $\psi = 0$ with $\psi > 0$ below (decreasing y). The crest of one wave corresponds to $\phi = 0$.

If we take

$$z = x + \mathrm{i}y \tag{2.1}$$

and

$$\chi = \phi + \mathrm{i}\psi \tag{2.2}$$

then this problem can be solved if we assume a suitable analytic expansion of z in terms of χ, as was done in Hogan (1980, 1981). The wavelength λ is taken to equal 2π, and so the wavenumber k is unity.

Thus we assume

$$z = -\frac{\chi}{c} + \mathrm{i} \sum_{n=1}^{\infty} \frac{1}{n} a_n \exp \frac{\mathrm{i}n\chi}{c}, \tag{2.3}$$

where the a_n are real coefficients. This form of the solution is periodic in ϕ and gives a uniform flow with speed c to the left at great depths. The a_n are determined from Bernoulli's equation at the free surface $y = \eta$, namely

$$\tfrac{1}{2}q_s^2 + g\eta + \frac{S}{R} = \text{constant}, \tag{2.4}$$

where q_s denotes the speed of surface particles, g is the acceleration due to gravity, S is the surface-tension coefficient divided by the density, and R is the radius of curvature. One method of obtaining values for the a_n is to assume a power-series expansion in terms of the wave amplitude h, defined as half the vertical distance between the wave crest and wave trough. In Hogan (1980) the assumption was made that

$$a_n = \sum_{k=0}^{\infty} \alpha_{nk} h^{n+2k} \quad (n = 1, 2, \ldots). \tag{2.5}$$

The coefficients α_{nk} were obtained by substituting (2.3) and (2.5) into (2.4), equating like powers of h and truncating. The Fourier coefficients a_n were then recovered by summing the series using Padé approximants. However, the form assumed in (2.5) leads to singularities in the α_{nk} at wavelengths given by

$$\kappa = \frac{1}{n} \quad (n = 2, 3, \ldots), \tag{2.6}$$

where

$$\kappa = \frac{4\pi^2 S}{\lambda^2 g}. \tag{2.7}$$

These are known as Wilton's (1915) ripples. Physically this corresponds to demanding a steady solution for wavelengths at which triad resonance, and hence possible wave generation, is known to occur. It turns out that several different steady solutions are possible at each singular value of κ, as shown by Wilton (1915), but this involves a different ordering of the coefficients a_n. Thus at $\kappa = \frac{1}{2}$ Hogan (1981) took

$$a_n = \sum_{j=1}^{\infty} \hat{\alpha}_{nj} h^{j-1+\mathrm{int}\,(\frac{1}{2}(n+1))} \quad (n = 1, 2, \ldots), \tag{2.8}$$

where int (p) means the integral part of p.

Other solution procedures (e.g. Chen & Saffman 1979, 1980) have avoided the assumption of series expansion, and this has led to the conclusion that several families of gravity–capillary waves exist at *all* values of κ. In this paper we concentrate on the members of families generated by expansions (2.5) and (2.8).

3. Method of solution

The time t taken for a particle to travel along the streamline $\psi = \psi_c$ from the point $\phi = \phi_1$ to $\phi = \phi_2$ is given by

$$t = \int_{\phi_1}^{\phi_2} \left|\frac{\mathrm{d}z}{\mathrm{d}\chi}\right|^2_{(\psi=\psi_c)} \mathrm{d}\phi \tag{3.1}$$

(see e.g. Longuet-Higgins 1979, §3). Then the trajectory of the particle in a frame of reference stationary with respect to great depths is given by

$$X = x + ct, \quad Y = y. \tag{3.2}$$

Thus from (2.3) we have

$$\begin{aligned} \left|\frac{\mathrm{d}z}{\mathrm{d}\chi}\right|^2 &= \frac{1}{c^2}\left|1 + \sum_{n=1}^{\infty} a_n \exp\frac{in\chi}{c}\right|^2 \\ &= \frac{1}{c^2}\left\{f_0(\psi) + 2\sum_{n=1}^{\infty} f_n(\psi)\cos\frac{n\phi}{c}\right\}, \end{aligned} \tag{3.3}$$

where

$$f_0(\psi) = 1 + \sum_{m=1}^{\infty} a_m^2 \exp\left(-\frac{2m\psi}{c}\right) \tag{3.4}$$

and

$$f_n(\psi) = \exp\left(-\frac{n\psi}{c}\right)\left[a_n + \sum_{m=1}^{\infty} a_m a_{m+n} \exp\left(-\frac{2m\psi}{c}\right)\right] \quad (n = 1, 2, \ldots). \tag{3.5}$$

Now it is a simple matter to calculate the total time T taken to complete one orbit, from the point $\phi_1 = 0$ to $\phi_2 = 2\pi c/k$ along the streamline $\psi = \psi_c$. From (3.1) and (3.3) we find

$$\frac{cT}{\lambda} = f_0(\psi_c). \tag{3.6}$$

From Hogan (1984), the time-averaged horizontal drift velocity U is given by

$$\frac{U}{c} = 1 - \frac{1}{f_0(\psi_c)}, \tag{3.7}$$

and the distance $[X]$ through which a particle moves is given by

$$\frac{[X]}{\lambda} = f_0(\psi_c) - 1. \tag{3.8}$$

κ	h	Type	$[X]/\lambda$	cT/λ	U/c
0	0.4431	g	0.287	1.287	0.223
0.000075	0.4365	g	0.282	1.282	0.220
0.0075	0.3545	g	0.15125	1.15125	0.13138
0.3	0.15	g	0.029210	1.029210	0.028381
0.4	0.11	g	0.031377	1.031377	0.030422
0.5	0.2	g	0.224728	1.224728	0.183492
0.5	0.2	c	0.072964	1.072964	0.068002
0.5	0.5450	c	0.622562	1.622562	0.383691
0.8	0.7243	c	0.899935	1.899935	0.473666
1.0	0.8069	c	1.028947	2.028947	0.507133
5.0	1.4846	c	2.782856	3.782856	0.735649
10.0	1.7468	c	3.971618	4.971618	0.798858
∞	2.2926	c	7.995564	8.995564	0.888834

TABLE 1. Parameters for complete orbits of surface particles in gravity–capillary waves

In table 1 we give these global properties $[X]/\lambda$, cT/λ and U/c for surface particles of various waves considered in Hogan (1980, 1981). The key quantity $f_0(\psi)$ was calculated by truncation and direct summation of the converged Fourier coefficients used in those papers. The table divides naturally into two parts, according to the type of wave. Gravity-like waves, denoted by the letter g, have a phase speed that increases with amplitude from the linear case, whereas capillary-like waves, denoted by the letter c, have a phase speed that decreases with increasing amplitude. This crude distinction is sufficient for our present purposes, but does not imply that waves of the same type belong to the same family.

Several features of this table require comment. The values given for $\kappa = 0$, $h = 0.4431$ differ from those given by Srokosz (1981). He found $[X]/\lambda = 0.375$, $cT/\lambda = 1.375$ and $U/c = 0.273$ for this highest wave. The difference lies in the fact that the series in (2.3) fails to converge satisfactorily for waves of limiting steepness. This has already been noted in Hogan (1980), and arises because the method was designed for use with waves influenced principally by surface tension. In this latter area, the convergence is excellent.

For $\kappa = 0.000075$ and 0.0075 the values of h used cannot be regarded as the highest possible physically but merely the best obtainable using this technique (the precise definition is given in Hogan 1980). Higher waves are to be anticipated, and, with that, possible higher values of the global trajectory properties. The same must also be said for the waves at $\kappa = 0.3$ and 0.4, as well as the gravity-like wave at $\kappa = 0.5$, $h = 0.2$. The capillary-like wave at $\kappa = 0.5$, $h = 0.2$ is included for comparison with the gravity-like wave with the same values of κ and h. It was possible to obtain higher capillary-like waves. In fact the limiting profile of a bubble enclosed in the trough was obtained in this case, although the value of h may have been underestimated (for full details see Hogan 1981). For $\kappa = 0.8$, 1.0, 5.0 and 10.0 the global properties are for surface particles of highest capillary-like waves. The results for pure capillary waves (infinite κ) are included for reference, having been obtained analytically in Hogan (1984). Clearly, increasing gravity has the effect of reducing the values of the global properties of the particle trajectories of waves strongly influenced by surface tension.

From table 1 we see that $[X] \approx 3$ cm for a wave with $\kappa = 0.0075$, $h = 0.3545$. On the other hand, for $\kappa = 10.0$, $h = 1.7468$ we have $[X] \approx 1.1$ cm. Thus since very steep short waves are often present when long nonlinear waves propagate, it is quite possible that short waves can contribute dramatically to the observed particle motion.

If we retain only those terms quadratic in the wave steepness, the results given by (3.6), (3.7) and (3.8) agree with the classical solution (see Lamb 1932, chap. 9).

4. Surface and subsurface particle trajectories

The trajectories of surface and subsurface particles are given by (3.2). This requires a knowledge of the time $t(\alpha)$ that a particle takes to move from (say) the crest, $\phi = 0$, to a point $\phi = \alpha c/k$, where $0 \leqslant \alpha \leqslant 2\pi$. We find this very simply from (3.1) and (3.3) to be given by

$$kct(\alpha) = \alpha f_0(\psi) + 2 \sum_{n=1}^{\infty} \frac{1}{n} f_n(\psi) \sin n\alpha. \tag{4.1}$$

From (2.3), (3.2) and (4.1) we find the coordinates (X, Y) of the orbit of the particle to be

$$X = \alpha(f_0(\psi) - 1) + \sum_{n=1}^{\infty} \frac{1}{n} (2f_n(\psi) - a_n \, \mathrm{e}^{-n\psi/c}) \sin n\alpha, \tag{4.2}$$

$$Y = -\frac{\psi}{c} + \sum_{n=1}^{\infty} \frac{1}{n} a_n \, \mathrm{e}^{-n\psi/c} \cos n\alpha, \tag{4.3}$$

where the $f_n(\psi)$ $(n = 0, 1, 2, \ldots)$ are given in (3.4) and (3.5). At great depths, $X \approx a_1 \, \mathrm{e}^{-\psi/c} \sin \alpha$, $Y + \psi/c \approx a_1 \, \mathrm{e}^{-\psi/c} \cos \alpha$, which gives

$$X^2 + \left(Y + \frac{\psi}{c}\right)^2 = a_1^2 \, \mathrm{e}^{-2\psi/c}. \tag{4.4}$$

This corresponds to circular paths of radius $a_1 \, \mathrm{e}^{-\psi/c}$ with centre $(0, -\psi/c)$.

We also evaluate the drift velocity U from (3.7), as a function of the mean displacement of a streamline from the surface. Thus if we define

$$\bar{y} \equiv \frac{1}{\lambda} \int_0^{\lambda} y \, \mathrm{d}x \tag{4.5}$$

then, using (2.3), it is straightforward to show that, for $\psi = \psi_c$,

$$\bar{y}_c = -\frac{\psi_c}{c} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} a_n^2 \exp\left(-\frac{2n\psi_c}{c}\right). \tag{4.6}$$

If we now set $\bar{y}_c = \bar{y}_0$ when $\psi_c = 0$, then the mean displacement of a streamline from the surface is given by

$$\frac{\bar{y}_c - \bar{y}_0}{\lambda} = -\frac{\psi_c}{c\lambda} - \frac{1}{4\pi} \sum_{n=1}^{\infty} \frac{1}{n} a_n^2 \left(1 - \exp\left(-\frac{2n\psi_c}{c}\right)\right), \tag{4.7}$$

where it must be remembered that $\lambda = 2\pi$.

The series in (4.2), (4.3) and (4.7) are summed by truncation and direct summation, exactly as for $f_0(\psi)$ in §3.

In view of the absence of a firm criterion for the highest wave when κ is small, it was decided to compare trajectory properties of waves with $\kappa = 0.0075$, $h = 0.35$ and with $\kappa = 0.000075$, $h = 0.35$. The results are given in table 2 to three significant figures. The surface particles of the wave most influenced by surface tension

κ	h	$\psi/c\lambda$	$[X]/\lambda$	U/c
0.0075	0.35	0	0.146	0.128
0.000075	0.35	0	0.145	0.127
0.0075	0.35	0.1	0.0251	0.0245
0.000075	0.35	0.1	0.0257	0.0250
0.0075	0.35	0.2	0.00655	0.00651
0.000075	0.35	0.2	0.00670	0.00666
0.0075	0.35	0.3	0.00182	0.00182
0.000075	0.35	0.3	0.00187	0.00186

TABLE 2. Comparison of surface and subsurface orbits for waves with $\kappa = 0.0075$, $h = 0.35$ and $\kappa = 0.000075$, $h = 0.35$

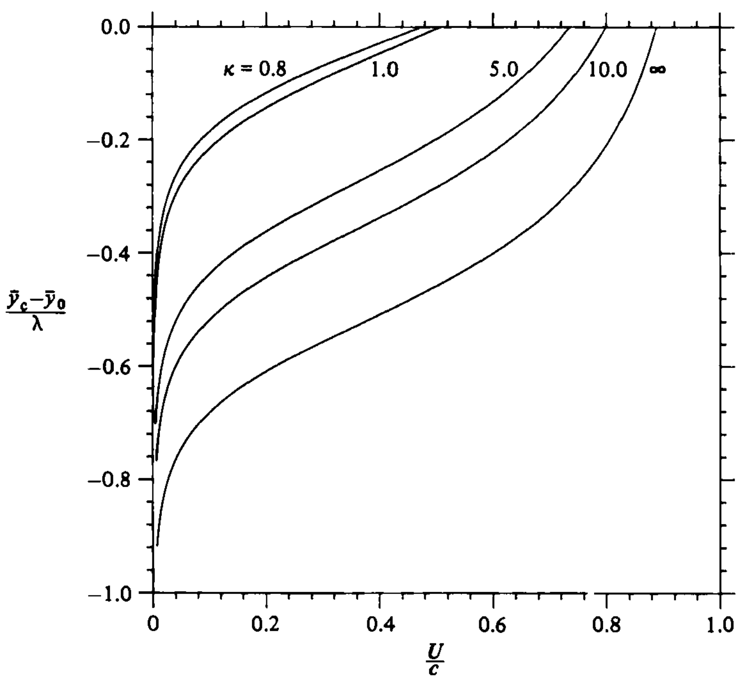

FIGURE 1. Drift-velocity ratio U/c for highest waves with $\kappa = 0.8$, 1.0, 5.0, 10.0 and κ infinite as a function of the mean displacement of fluid particles $(\bar{y}_c - \bar{y}_0)/\lambda$.

($\kappa = 0.0075$) travel further and, on average, faster than the wave with $\kappa = 0.000075$. But it is quite noticeable that the same wave's subsurface particles travel less far and, on average, slower. This can be explained by the fact that for $\kappa = 0.000075$ the first two Fourier coefficients a_1 and a_2 of the wave profile are actually larger than the corresponding coefficients for $\kappa = 0.0075$. The remainder, however, are smaller. Thus two harmonics whose influence is felt at greater depths have larger amplitudes, and so can be expected to give larger values for the trajectory properties. In fact for $\kappa = 0.000075$, $h = 0.35$ the set of coefficients $(a_1, a_2, a_3, a_4) = (0.28333,\ 0.17362,\ 0.12103, 0.08927)$, whereas for $\kappa = 0.0075$, $h = 0.35$ the set takes the values (0.27992, 0.17249, 0.12153, 0.09224). On the other hand, the difference at the surface is quite consistent with results in the rest of this paper, that is, an increase in surface tension tends to increase the properties of surface particles.

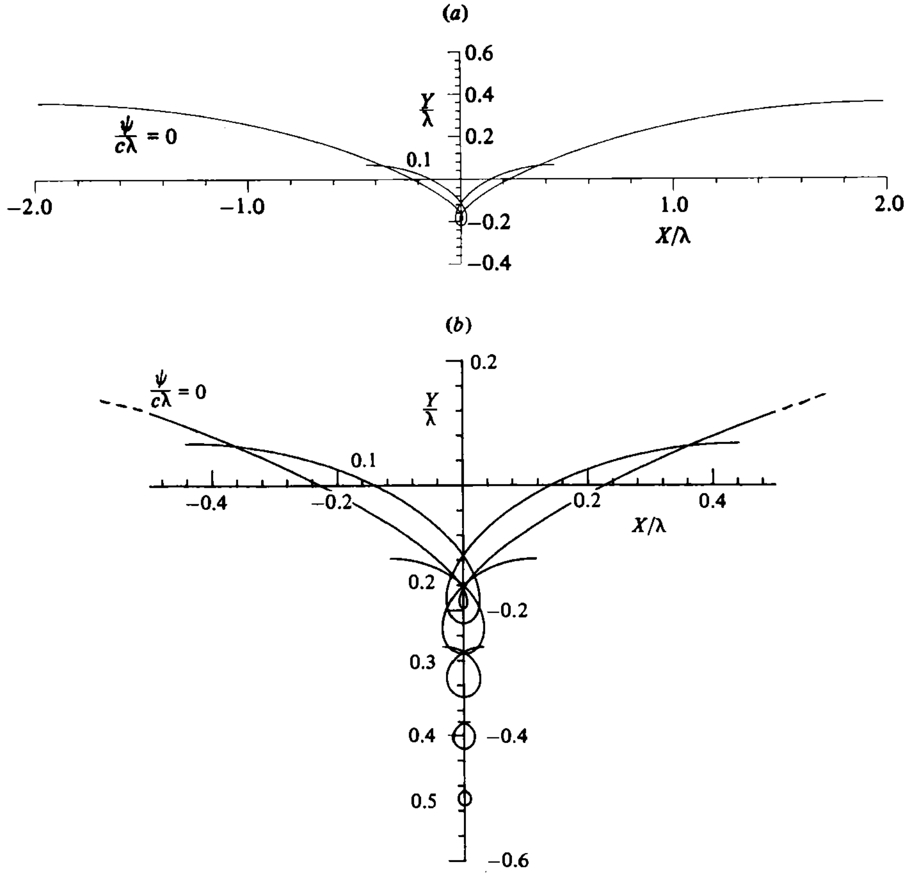

FIGURE 2. (*a*) Particle trajectories for $\kappa = 10.0$, $h = 1.7468$ along streamlines $\psi/c\lambda = 0, 0.1$. (*b*) Full trajectories for $\psi/c\lambda = 0.1, 0.2, 0.3, 0.4, 0.5$ together with part trajectory for $\psi/c\lambda = 0$.

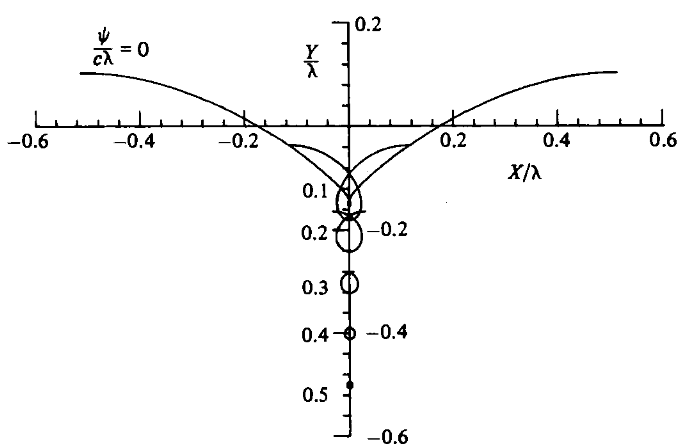

FIGURE 3. Particle trajectories for $\kappa = 1.0$, $h = 0.8069$ along streamlines $\psi/c\lambda = 0, 0.1, 0.2, 0.3, 0.4, 0.5$.

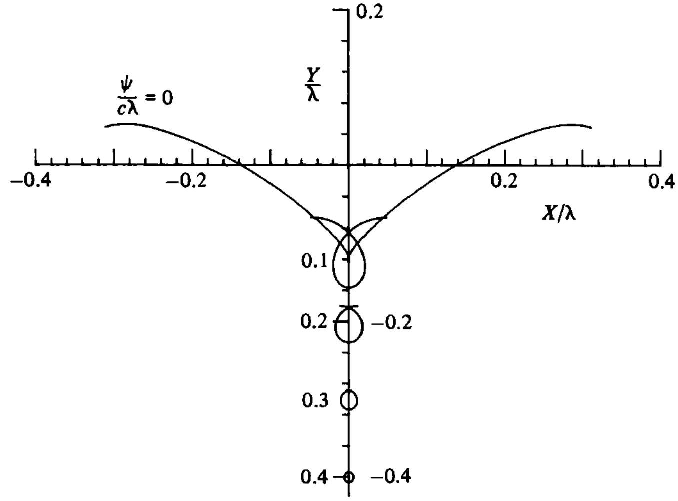

FIGURE 4. Particle trajectories for $\kappa = 0.5$, $h = 0.5450$ (capillary-like branch) along streamlines $\psi/c\lambda = 0, 0.1, 0.2, 0.3, 0.4$.

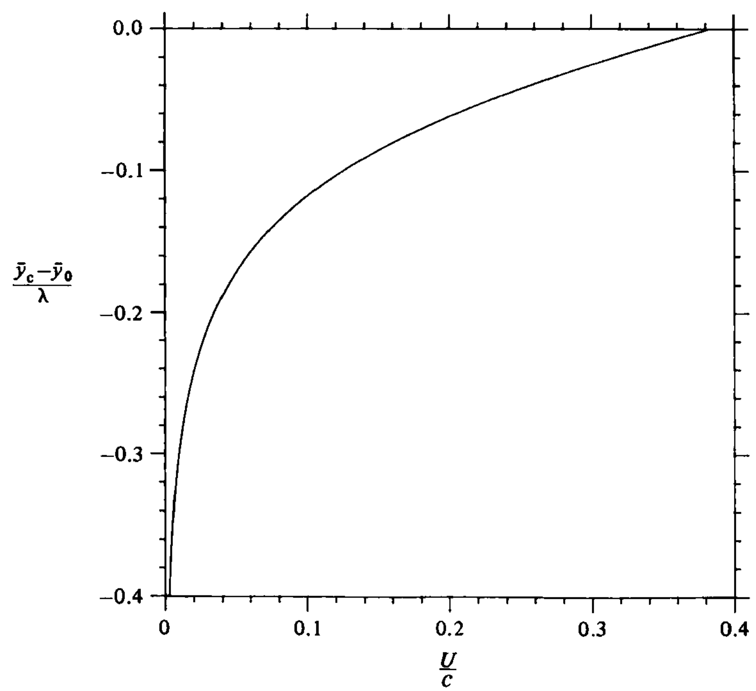

FIGURE 5. As figure 1, for $\kappa = 0.5$, $h = 0.5450$ (capillary-like branch).

In figure 1 we plot drift-velocity profiles for $\kappa = 0.8$, 1.0, 5.0, 10.0 and κ infinite. The highest wave and its streamlines are considered in each case. We note that gravity's influence need only be small in order to severely reduce the depth and extent of the pure capillary wave drift profile. Thus $\kappa = 0.8$ corresponds to a wavelength of nearly 2 cm in water, yet the surface-drift velocity ratio is nearly halved and the

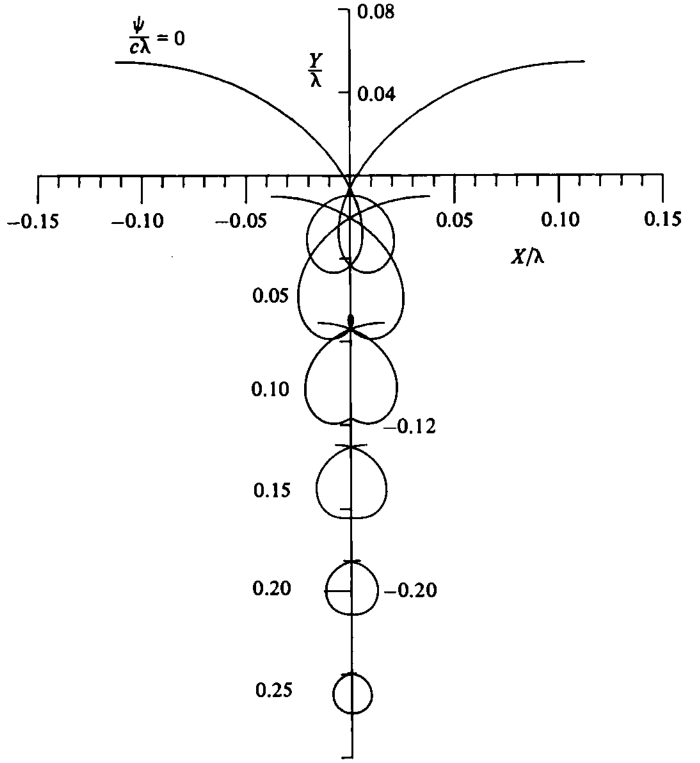

FIGURE 6. Particle trajectories for $\kappa = 0.5$, $h = 0.20$ (gravity-like branch) along streamlines $\psi/c\lambda = 0, 0.05, 0.10, 0.15, 0.20, 0.25$.

depth of penetration cut by two-thirds. The original wave profiles for $\kappa = 1.0, 5.0, 10.0$ and κ infinite are illustrated in figure 12 of Hogan (1980). The particle trajectories for the surface and subsurface particles of the highest wave with $\kappa = 10.0$ are illustrated in figures 2(*a*, *b*). This division is necessary for reasons of scale. Similar results hold for the highest wave with $\kappa = 5.0$. The results for $\kappa = 1.0$ are given in figure 3, with similar results holding for $\kappa = 0.8$. Figures 2 and 3 can be compared with figure 3 of Hogan (1984). Again we note that only a small departure from the gravity-free case is enough to change the character of the trajectory dramatically. For example, when $\kappa = 0.8$ the overall distance $[X]/\lambda$ travelled by a surface particle, is nearly nine times smaller than for pure capillary waves. In addition, no particle orbit is completely open, with the closed section always present near the trough. Consequently there is no cusp in any orbit, unlike the pure capillary case. We cannot, however, rule out such behaviour for finite $\kappa > 10.0$. In each case the surface particles spend a lot of their time near the wave crest, although for $\kappa = 0.8$ the time spent near the trough becomes significant (nearly 50%).

We now consider the particle trajectories of waves calculated in Hogan (1981). This work was concerned with nonlinear examples of Wilton's ripples, and in particular with the first ripple at $\kappa = 0.5$. It proved possible to derive a highest capillary-like wave, with $h = 0.5450$. This has a dip in its crest, but otherwise is very similar to capillary waves with larger values of κ. It is illustrated in figure 4 of Hogan (1981). The particle trajectories are given in figure 4 and the drift-velocity profile in figure 5.

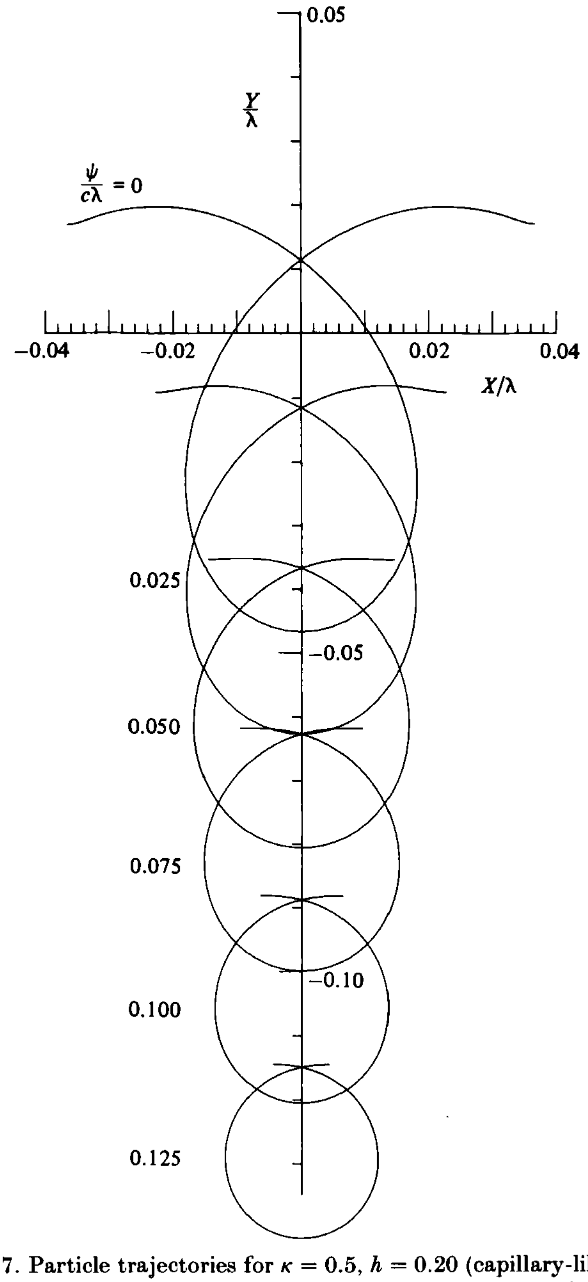

FIGURE 7. Particle trajectories for $\kappa = 0.5$, $h = 0.20$ (capillary-like branch) along streamlines $\psi/c\lambda = 0$, 0.025, 0.050, 0.075, 0.100, 0.125.

The trajectory clearly rises after the particle passes through a crest. Although a little difficult to discern on this scale, the tangent to the trajectory is horizontal at the crest.

The highest waveform of the gravity-like wave at $\kappa = 0.5$ is still unknown. Nevertheless, convergent profiles were obtained up to $h = 0.20$, which contained an elevation in the trough (Hogan 1981, figure 6). It is of interest to compare particle trajectories of these two types of waves, both with $\kappa = 0.5$ and $h = 0.20$, but otherwise

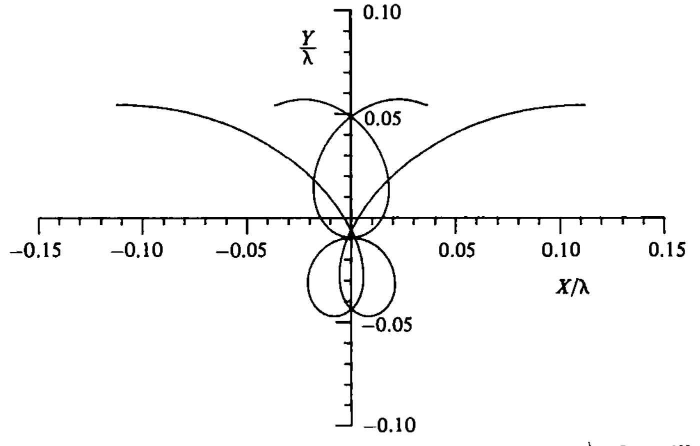

FIGURE 8. Free-surface particle trajectories for both gravity- and capillary-like waves at $\kappa = 0.5$, $h = 0.20$ (drawn with coincident troughs).

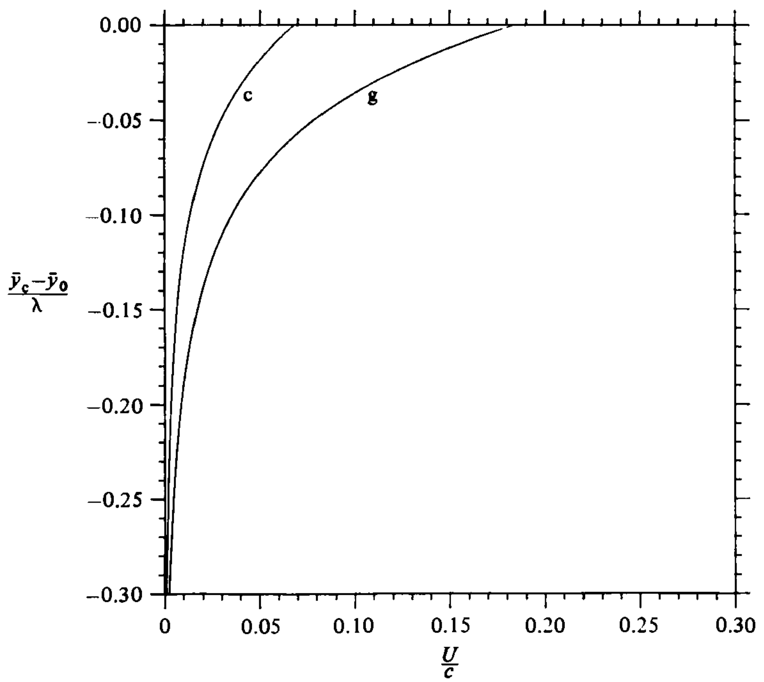

FIGURE 9. Drift-velocity ratios U/c as a function of the mean displacement of fluid particles $(\bar{y}_c - \bar{y}_0)/\lambda$ for both gravity- and capillary-like waves at $\kappa = 0.5$, $h = 0.20$.

distinct. The trajectories of the surface and subsurface particles of the gravity-like wave are illustrated in figure 6, with the capillary-like trajectories in figure 7. The free-surface trajectories are compared in figure 8 and the drift-velocity profiles in figure 9. Immediately it is clear that there is a great difference between the two types. In particular, the double-loop trajectory of the free-surface particles of the gravity-like waves is remarkable, with the form persisting for some subsurface particles. Nevertheless, at great depths it becomes circular. For capillary-like waves the trajectory rises after the particle at $\psi/c\lambda = 0.075$ passes a crest, but quickly regains its circular

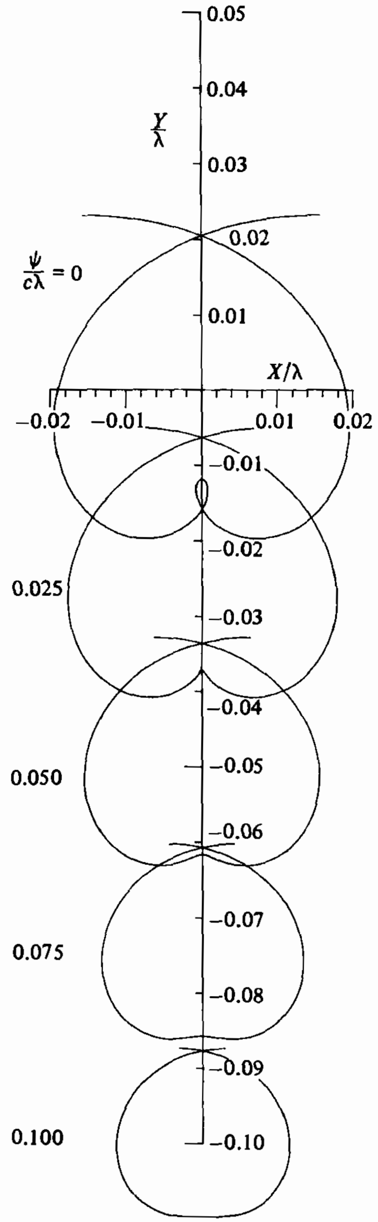

FIGURE 10. Particle trajectories for $\kappa = 0.4$, $h = 0.11$ along streamlines $\psi/c\lambda = 0, 0.025, 0.050, 0.075, 0.100$.

form at larger values of $\psi/c\lambda$. The difference is probably best highlighted in figure 8. Here the capillary-like wave has been drawn so that the y-coordinate of its trough (and hence its crest) coincides with that of the gravity-like wave (the mean levels of the two waves are not equal). It is noticeable how both the vertical and horizontal excursions of the surface particle in the gravity-like wave are larger than the capillary-like wave. The total time taken to cover both orbits is about the same, but the gravity-like wave particles travel nearly three times as fast. The trajectories of

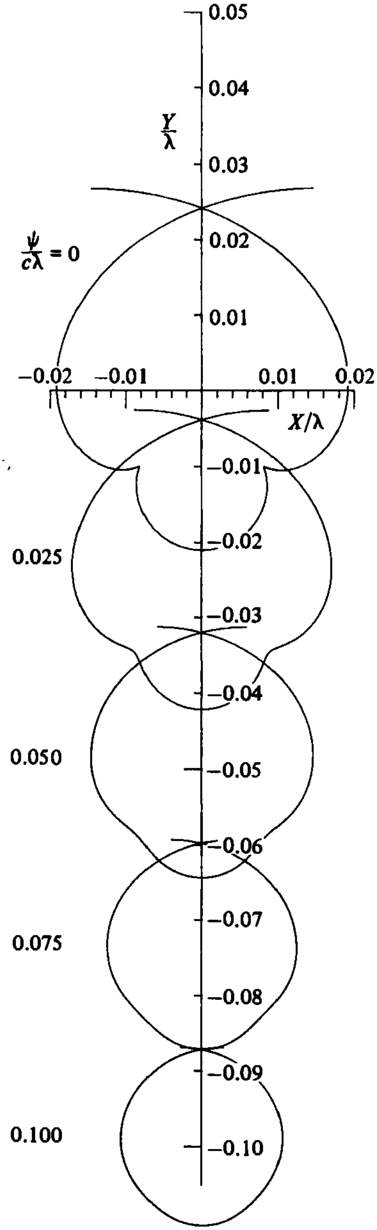

FIGURE 11. As figure 10, for $\kappa = 0.3$, $h = 0.15$.

particles in the highest gravity-like wave at $\kappa = 0.5$ remain a matter of speculation, but the double-loop nature can be expected to remain, particularly since, with a crest in its trough and the subtroughs deepening, the wave may look very like two waves side by side.

In Hogan (1981) the gravity-like wave at $\kappa = 0.5$ was compared with waves at $\kappa = 0.3$ and 0.4 (Hogan 1981, figures 1 and 2). The steep waves at $\kappa = 0.4$ have an elevation in their troughs. But at $\kappa = 0.3$ the steep waves have two elevations in their troughs and are between two different Wilton-ripple wavelengths ($\kappa = \frac{1}{4}$ and $\frac{1}{3}$). In figure 10 we present the particle trajectories for waves at $\kappa = 0.4$ and $h = 0.11$. This is very similar to figure 6. Both waves considered in figure 6 and figure 10 will contain a trajectory cusped in the trough. The exact values of $\psi/c\lambda$ have not been determined. It was not possible to obtain higher waves owing to convergence difficulties, but the double-loop nature is expected to persist and become more noticeable as the waveheight increases. In figure 11 the trajectories for $\kappa = 0.3$ and $h = 0.15$ are given. The orbit of the surface particles now contain two cusp-like features which correspond to the elevations in the trough. For higher waves it could be expected that the cusps will become subloops, which may subsequently overwhelm the main loop. The surface trajectories in figures 10 and 11 are also notable, since the horizontal excursion of the particle is actually less than its vertical excursion, in contrast with the other trajectories calculated in this paper.

5. Discussion

We have presented results for the particle trajectories of nonlinear water waves influenced by both surface tension and gravity. When the former is dominant, the results are similar to those obtained by the present author for gravity-free propagation (Hogan 1984). However, the influence of gravity need only be very small to reduce the scale of those results dramatically. When surface-tension forces are weak the trajectory properties are very similar to the pure-gravity case for waves of the same steepness. When the forces are approximately equal, waves with the same length and height but belonging to different families of Wilton's ripples are seen to possess completely different trajectory properties. This could be used as a way of distinguishing between them. Still further waves, with elevations in their troughs, are shown to possess extraordinary trajectories with subloops on a main circular loop (but rather mundane drift-velocity profiles). In summary, the effect of increasing surface tension is to produce an increase in the relative horizontal distance $[X]/\lambda$ travelled by a surface particle in an orbit and generally to increase the magnitude and extent of the time-averaged drift velocity ratio.

On other families (branches), gravity–capillary waves are known to possess even more elevations in their troughs (Chen & Saffman 1980, figures 9 and 10), and so presumably may have many looped structures in their particle trajectories.

As in Hogan (1984), viscosity and surface-tension gradients have been ignored in this study. Similar remarks apply in this case as then, with the possible difference that the effects will become less important as surface tension decreases in relation to gravity.

As explained in §3, very short steep ripples can have similar absolute particle displacements to longer nonlinear gravity waves. This may have implications for the observation of particle motions in steep gravity waves. Also the wind-drift current may well be modified in the presence of steep capillary waves. These results also have implications for mass transport of surface particles in partly filled horizontal pipes, where capillaries may be generated.

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